NAG Toolbox for MATLAB

g01fm

1 Purpose

g01fm returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic, via the function name.

2 Syntax

```
[result, ifail] = g01fm(p, v, ir)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery 1984 and Winer 1970).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left(r \int_{-\infty}^{\infty} \phi(y)(\Phi(y) - \Phi(y - qx))^{r-1}\, dy \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \qquad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \qquad \text{and} \qquad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

For a given probability p_0, the deviate q_0 is found as the solution to the equation

$$P(q_0; v, r) = p_0, \tag{1}$$

using c05az . Initial estimates are found using the approximation given in Lund and Lund 1983 and a simple search procedure.

4 References

Lund R E and Lund J R 1983 Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32 (2)** 204–210

Montgomery D C 1984 *Design and Analysis of Experiments* Wiley

Winer B J 1970 *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **p – double scalar**

The lower tail probability for the Studentized range statistic, p_0.

Constraint: $0.0 < \mathbf{p} < 1.0$.

2: **v – double scalar**

v, the number of degrees of freedom.

Constraint: $\mathbf{v} \geq 1.0$.

3: **ir – int32 scalar**

r, the number of groups.

Constraint: $\mathbf{ir} \geq 2$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01fm may return useful information for one or more of the following detected errors or warnings.

If on exit **ifail** $= 1$, then g01fm returns 0.0.

ifail $= 1$

On entry, $\mathbf{p} \leq 0.0$,
or $\mathbf{p} \geq 1.0$,
or $\mathbf{v} < 1.0$,
or $\mathbf{ir} < 2$.

ifail $= 2$

The function was unable to find an upper bound for the value of q_0. This will be caused by p_0 being too close to 1.0.

ifail $= 3$

There is some doubt as to whether full accuracy has been achieved. The returned value should be a reasonable estimate of the true value.

7 Accuracy

The returned solution, q_*, to equation (1) is determined so that at least one of the following criteria apply.

(a) $|P(q_*;v,r) - p_0| \leq 0.000005$

(b) $|q_0 - q_*| \leq 0.000005 \times \max(1.0, |q_*|)$.

8 Further Comments

To obtain the factors for Duncan's multiple-range test, equation (1) has to be solved for p_1, where $p_1 = p_0^{r-1}$, so on input **p** should be set to p_0^{r-1}.

9 Example

```
p = 0.95;
v = 10;
ir = int32(5);
[result, ifail] = g01fm(p, v, ir)
```

```
result =
    4.6543
ifail =
           0
```